Homework problems for October 10, 2016

Exercise 1: Show that if $Z_1, \ldots, Z_n$ are uncorrelated random variables and $\alpha_t, \beta_t \in \mathbb{R}$ for $t = 1, \ldots, n$, then

$$\operatorname{Cov}\left(\sum_{t=1}^{n} \alpha_t Z_t, \sum_{s=1}^{n} \beta_s Z_s\right) = \sum_{t=1}^{n} \alpha_t \beta_t \operatorname{Var}(Z_t).$$

For the following exercises, consider the linear model

$$y = X\beta + u,$$

where

1) $\mathbb{E}[u] = 0$.
2) $\mathbb{E}[uu'] = \sigma^2 I_n$.
3) X is an $n \times k$ non-random matrix
4) $\operatorname{rank}(X) = k$.

Exercise 2: Calculate $\mathbb{E}[y]$ and $\mathrm{VC}[y]$.

Exercise 3: Suppose now that assumption 4) is violated. Show that there exists a vector $\beta^* \in \mathbb{R}^k$, $\beta^* \neq \beta$, such that $y = X\beta^* + u$. For estimating β, why is this a problem? For estimating $\mathbb{E}[y]$, why isn't this a problem?

Exercise 4: For positive integers $m < n$, let $y_t = a + u_t$, if $t = 1, \ldots, m$, and let $y_t = a + b + u_t$, if $t = m + 1, \ldots, n$, where $a, b \in \mathbb{R}$ are fixed, unknown constants and $\mathbb{E}[u_t] = 0$, $\mathbb{E}[u_t^2] = \sigma^2$ and $\mathbb{E}[u_t u_s] = 0$ if $t \neq s$. Find a matrix X and a vector β such that the random vector $y = (y_1, \ldots, y_n)'$ follows the linear model

$$y = X\beta + u,$$

and verify the assumptions $1 - 4$.